CSC263 Tutorial #10

MSTs

March 24, 2023

Things covered in this tutorial

- ⋆ What's an MST?
- ⋆ How to find an MST?
- ⋆ How to prove that a solution is an optimal (i.e. minimum/maximum weight) solution?

MSTs

Task: What does MST stand for?

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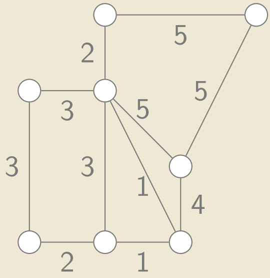
Answer: Minimum Spanning Tree. (Or sometimes, *Maximum* Spanning Tree!)

MSTs

Task: Find a MST (in both definitions!) on the following graph. Are there multiple MSTs?



Some properties of trees

Task: List any properties of trees that you know of!

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Trees satisfy the following:

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Trees satisfy the following:

- ⋆ A tree is connected.
- ⋆ Between any two vertices u and v in a tree, there is a *unique* path from u to v.
- ⋆ If a tree has N vertices, then it has $N - 1$ edges.
- ⋆ A graph is a tree iff it is connected and has no cycles.

Cyclers and Trees

A cycler is a set S of vertices in G such that every cycle contains at least one edge in S.

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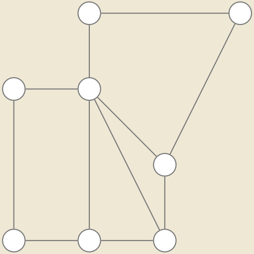
Task: Identify a cycler in the below graph, taking as few edges as you can. What is its *complement*?



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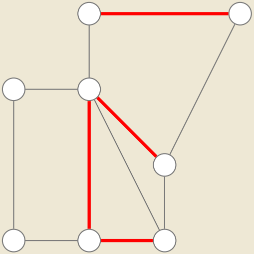

Cycler

Cyclers and Trees

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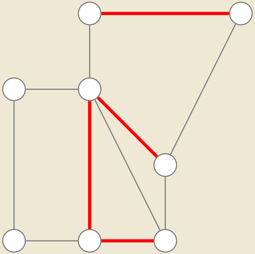
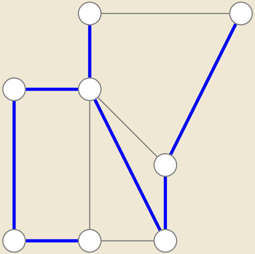


Cycler



Complement

Notice anything about the complement?

Cyclers and Trees

Theorem

A set S of edges is a cycler iff its complement S^c has no cycles.

Cyclers and Trees

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Proof:

Seriously though: prove it!
A cycler is a set S of vertices in G such that every cycle contains at least one edge in S.

Cyclers and Trees

Theorem

A set S of edges is a cycler iff its complement S^c has no cycles.

Theorem

Assuming G is connected and has positive edge weights, a set S of edges is a minimum cycler iff its complement S^c is ...

Task: Fill in the blank!

Cyclers and Trees

Theorem

A set S of edges is a cycler iff its complement S^c has no cycles.

Theorem

Assuming G is connected and has positive edge weights, a set S of edges is a minimum cycler iff its complement S^c is a maximum spanning tree.

Task: Fill in the blank!

Cyclers and Trees

Theorem

Assuming G is connected and has positive edge weights, a set S of edges is a minimum cycler iff its complement S^c is a maximum spanning tree.

Proof:

Cyclers and Trees

Theorem

Assuming G is connected and has positive edge weights, a set S of edges is a minimum cycler iff its complement S^c is a maximum spanning tree.

Task: Complete the proof!

($\Rightarrow$) Suppose S is a minimum cycler. Then:

- $\star$ S^c must be a spanning tree, because .... Thus any maximum spanning tree T must have $w(T) \geq w(S^c)$.
- $\star$ $w(S^c) \geq w(T)$ for any maximum spanning tree T, because ....

Since $w(S^c) = w(T)$ for any maximum spanning tree T, it follows S^c is a maximum spanning tree.

($\Leftarrow$) Suppose S^c is a maximum spanning tree. Then:

- $\star$ ...
- $\star$ ...

Since ...

Multiple MSTs??

Task: Find as many MinimumSTs as possible on the following graph! Then do the second tutorial activity.