CSC263 Tutorial #2

Heaps

January 20, 2023

Things that will be covered in this tutorial

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- ⋆ What's a priority queue?
- ⋆ What's a (max-)heap? How can I access elements of heaps?
- ⋆ What's a "ternary max-heap"?

Priority Queues

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Question: What can a priority queue be used for?
Answer: Things like hospital triage, or your workload.

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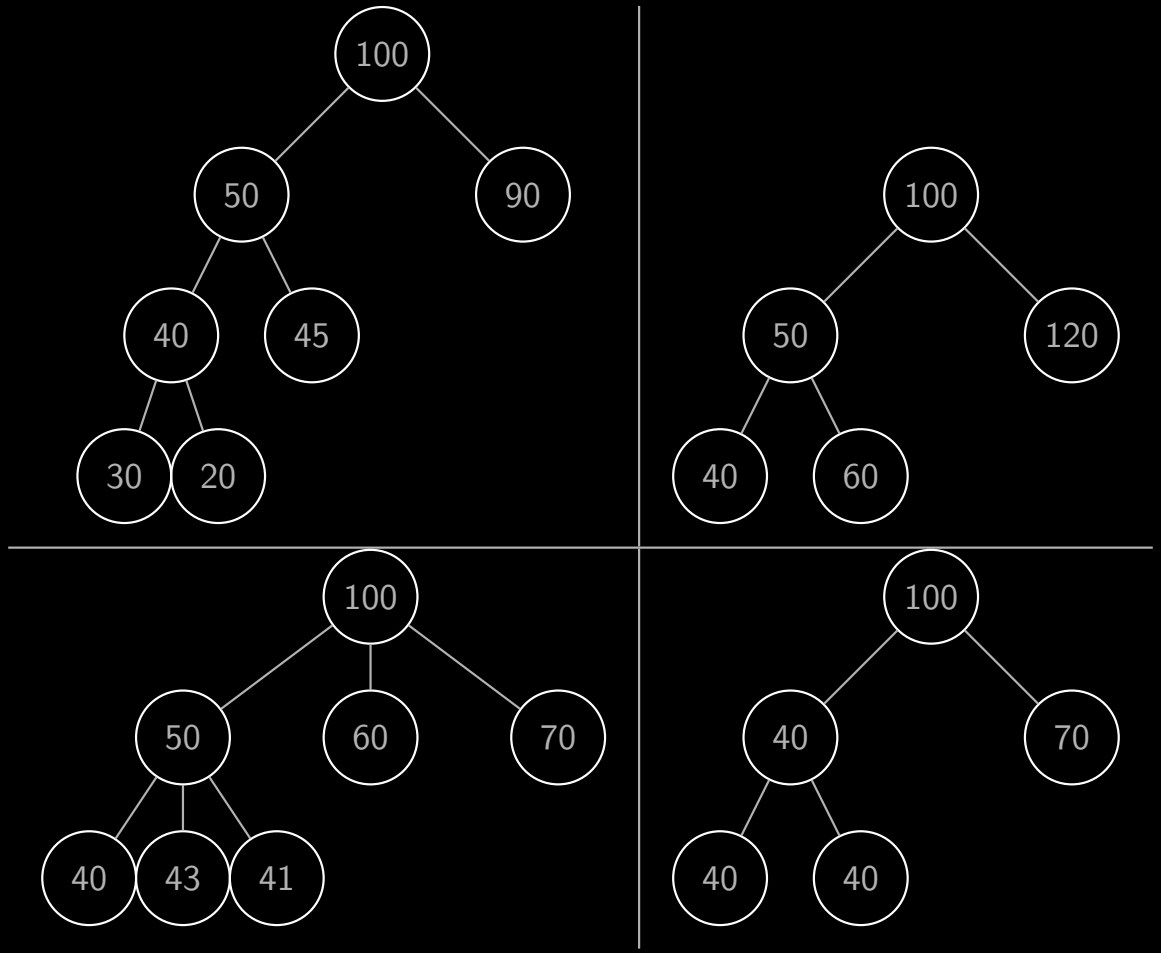
Question: Which of the following is a binary max-heap?



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Hint: There are two correct answers, depending on the assumptions you make.

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Hint: There is only one correct answer this time.

Ternary Heaps

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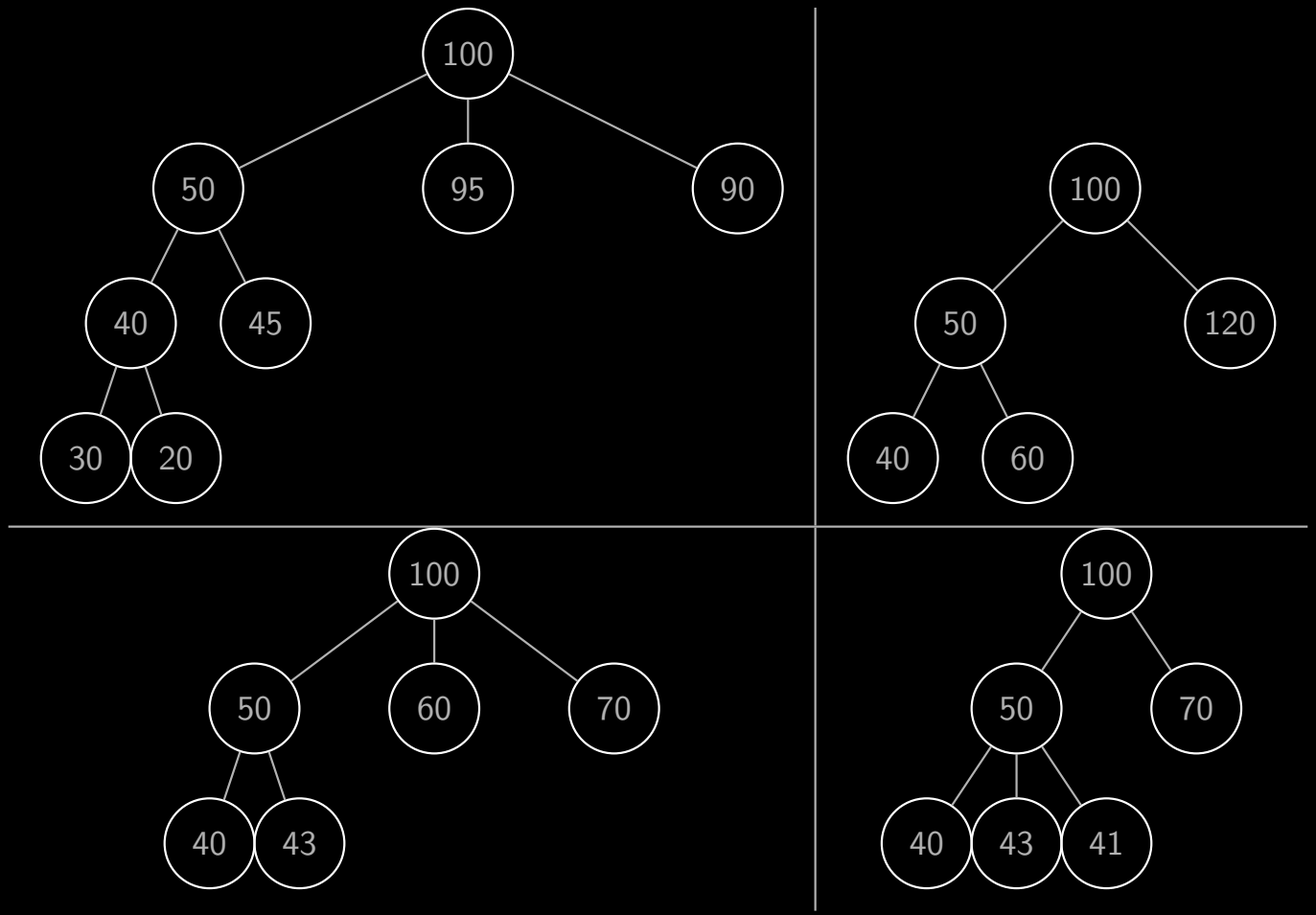
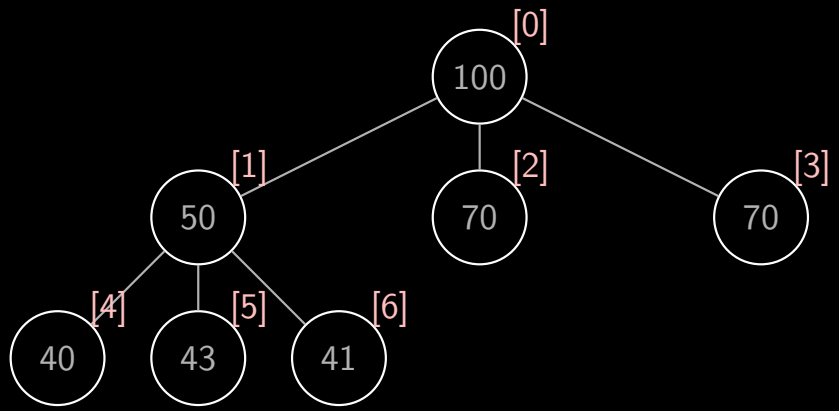
Like binary heaps, but with three children (instead of two)!
Question: Which of the following is a ternary max-heap?



Ternary Heaps

Ternary Heaps

Ternary Heaps are represented with arrays.



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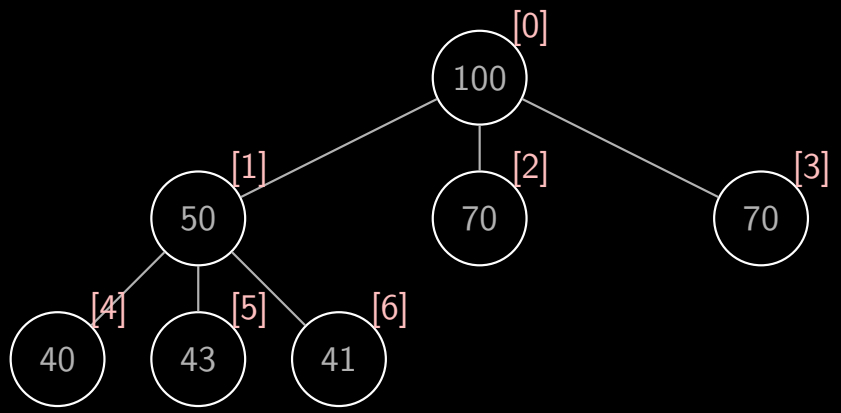

Question:

- ⋆ Which index is the left child of [0]?
- ⋆ Which index is the middle child of [1]?
- ⋆ Which index is the parent of [2]?
- ⋆ Which index is the parent of [6]?

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Tutoiral Question: How to check when an array is a ternary heap?

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```
ArrayIsBinaryMaxHeap(A):
    n = len(A)
    for i in 0..(A//2-1):
        if A[i] < A[2i] or A[i] < A[2i+1]:
            return False
    return True
```

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```
ExtractMax(A):
    n = len(A)
    swap A[0] with A[n]
    max = A.pop()
    BubbleDown(A[0])
    return max
```

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```
Insert(A, x, k):
    x.priority = k
    A.insert(x)
    BubbleUp(x)
```