CSC263 Tutorial #9

DFS and Bipartite Graphs

March 17, 2023

Things covered in this tutorial

- ⋆ What's a bipartite graph?
- ⋆ How can I check whether a graph is bipartite using DFS?

Bipartite graphs

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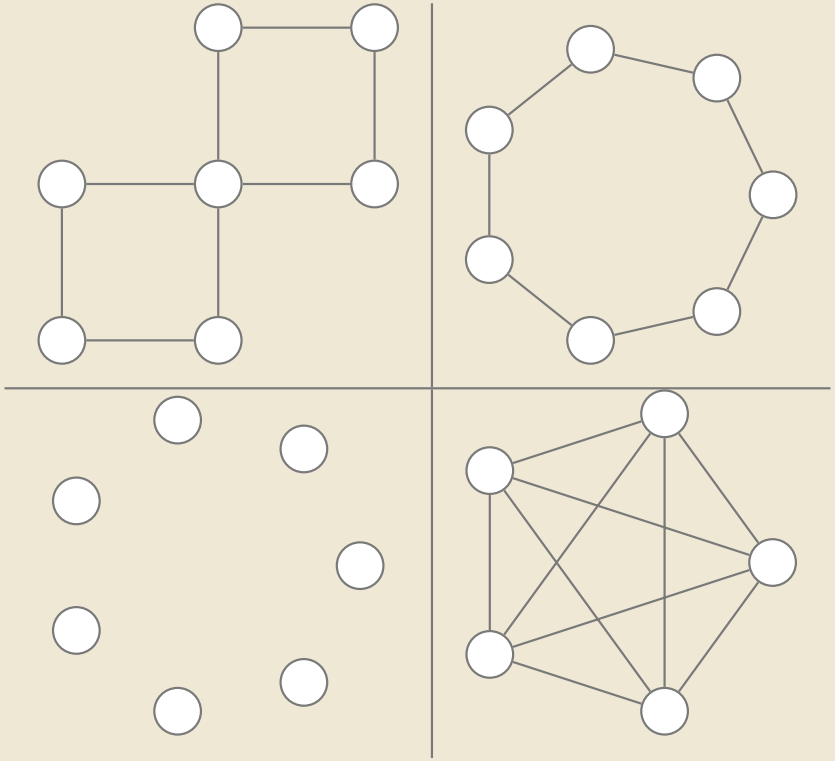
- $V_1 \cup V_2 = V$, $V_1 \cap V_2 = \emptyset$. In other words, each vertex belongs to either V_1 or V_2, but not both.
- Every edge must have one endpoint in V_1 and the other in V_2. In other words, every edge must "cross" from V_1 to V_2.

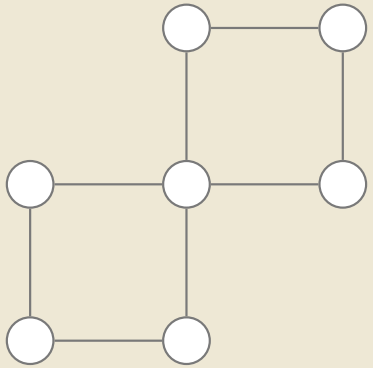
Bipartite graphs

Task: Which of the following graphs are bipartite?



Bipartite Graphs

Colour the vertices to represent V_1 (black) and V_2 (white).



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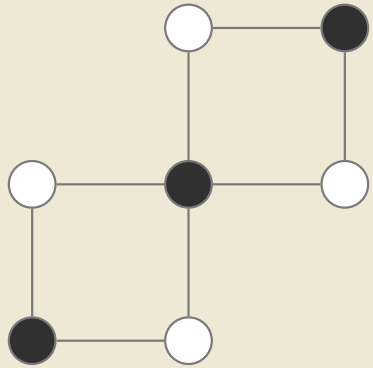

Bipartite Graphs

How to check if a graph is bipartite?

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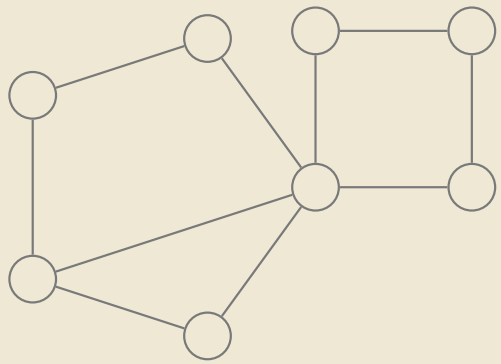
How to check if a graph is bipartite? DFS, but colour the nodes with alternating colours.



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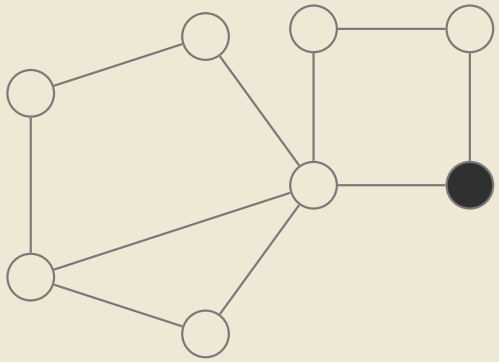

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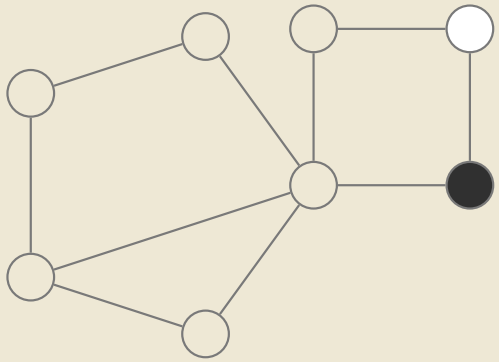

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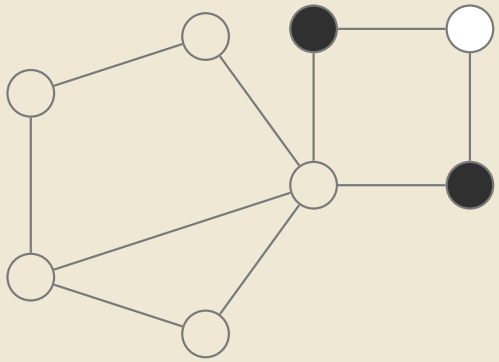

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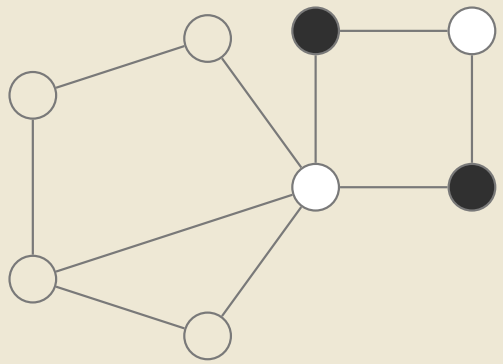

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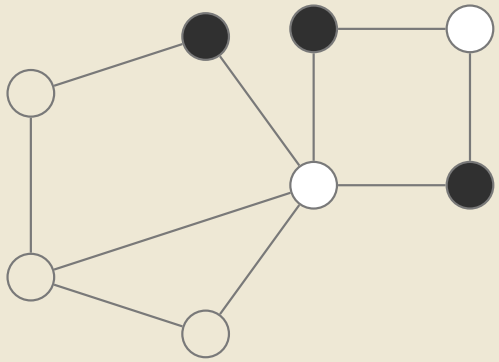

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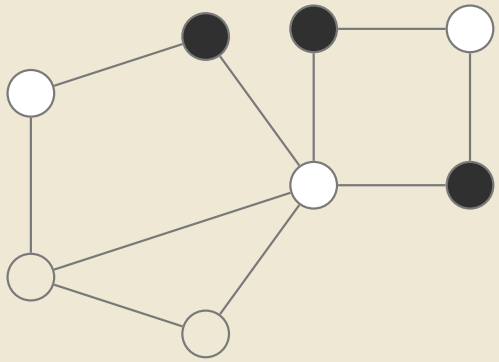

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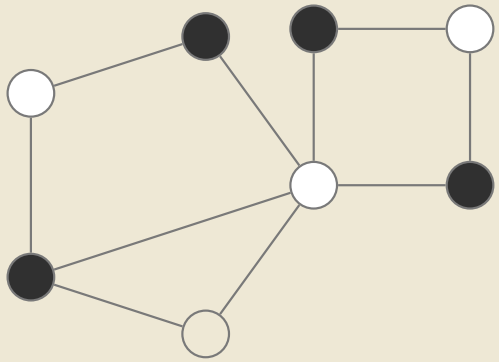

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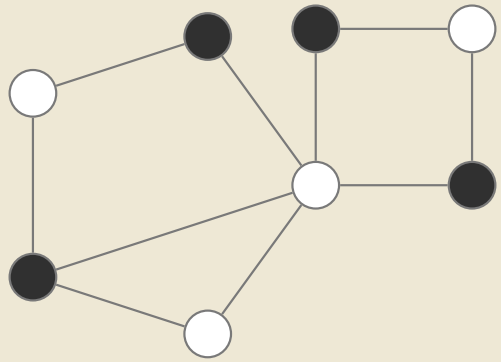

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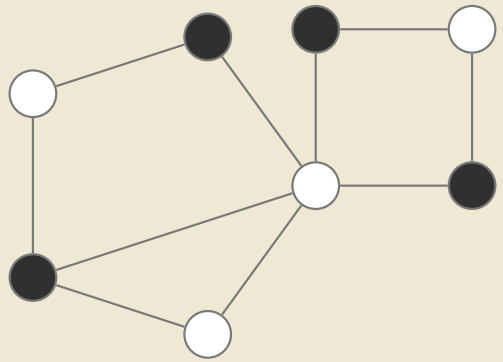

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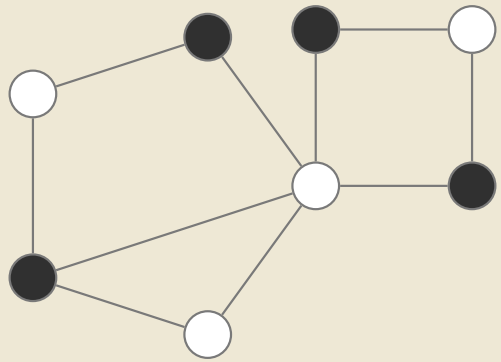
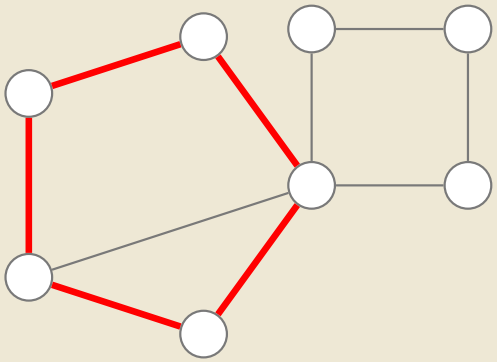


Question: Is this graph bipartite?
Tutorial Activity: Write pseudocode for a function `check_bipartite` to check whether a graph is bipartite!

Bipartite Graph Theorem

Theorem

A graph is bipartite iff it has no odd cycles.



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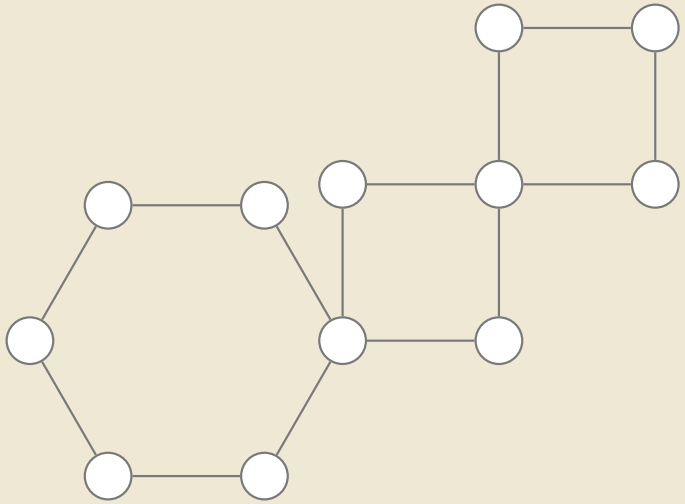

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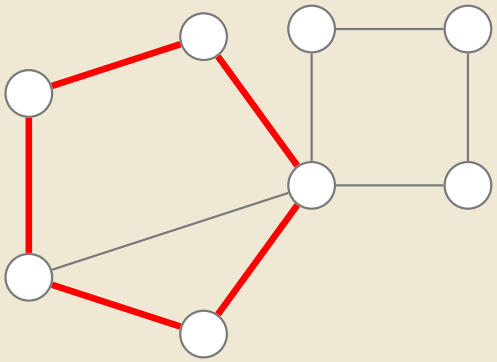

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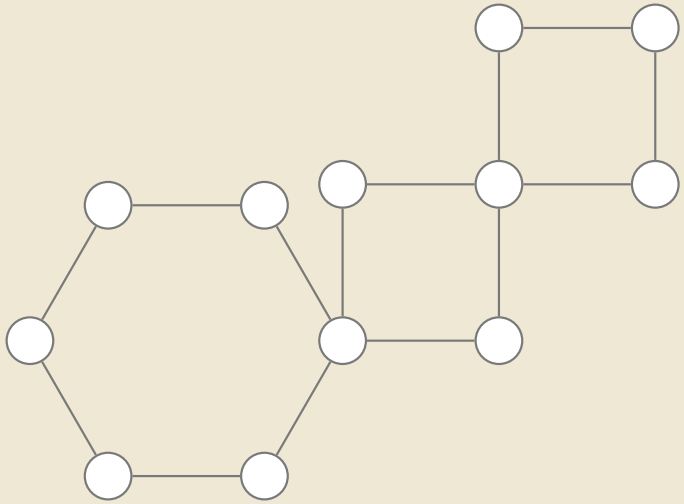

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($\Rightarrow$) We prove the contrapositive. Suppose our graph has an odd cycle. We want to show it is not bipartite.

Question: Why can't a graph with an odd cycle be bipartite?

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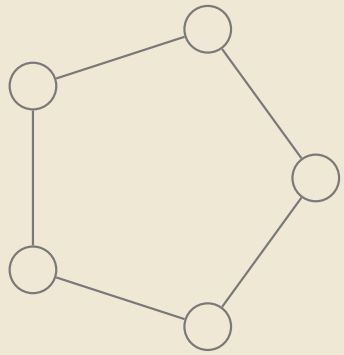

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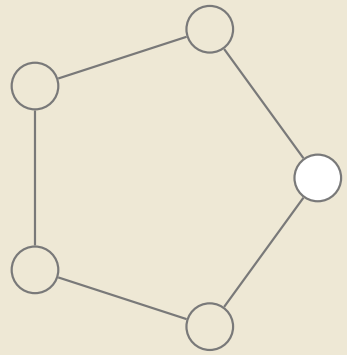

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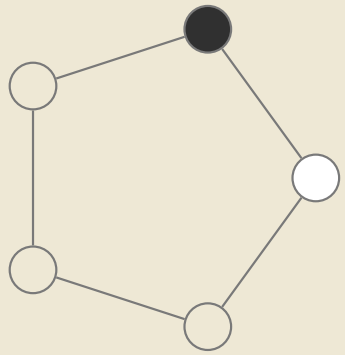

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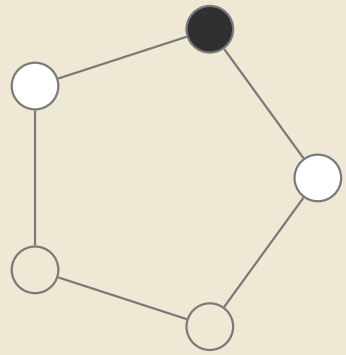

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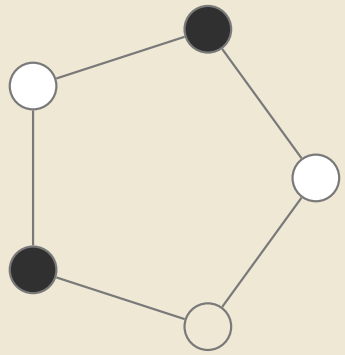

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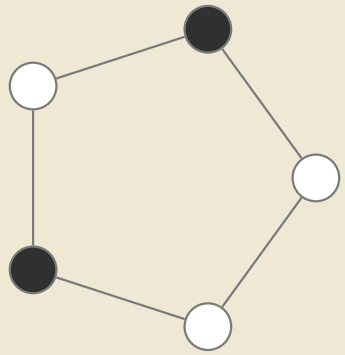

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Answer: `check_bipartite`!

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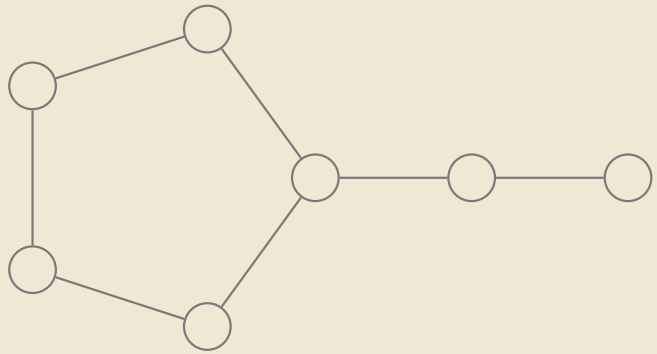

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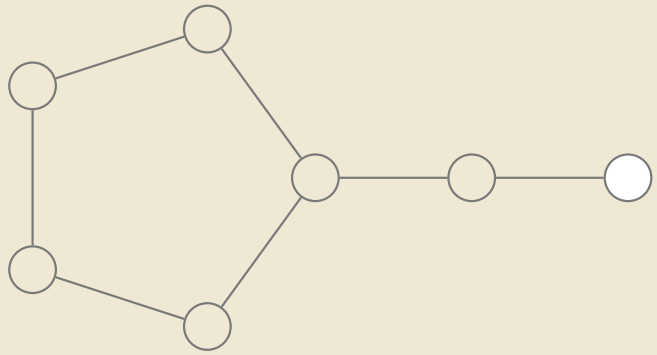

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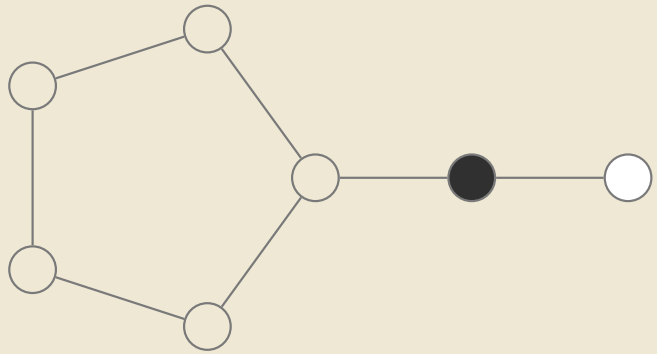

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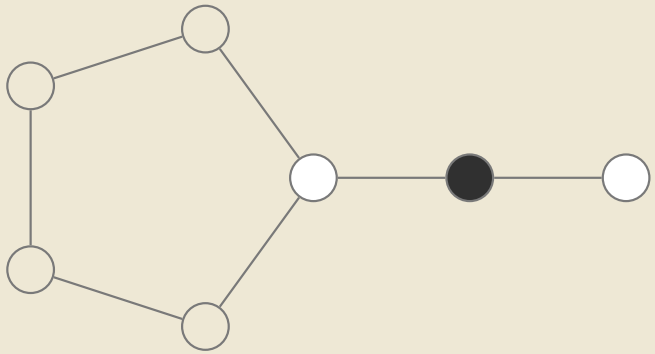

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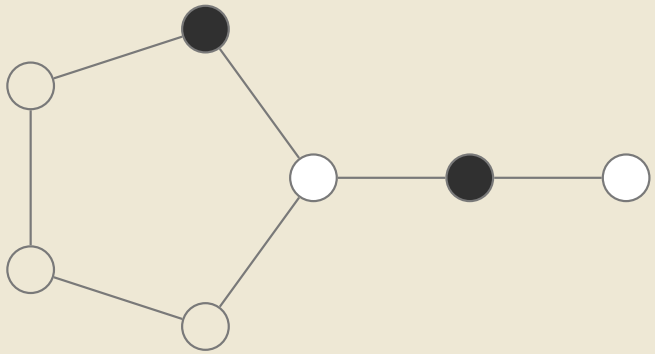

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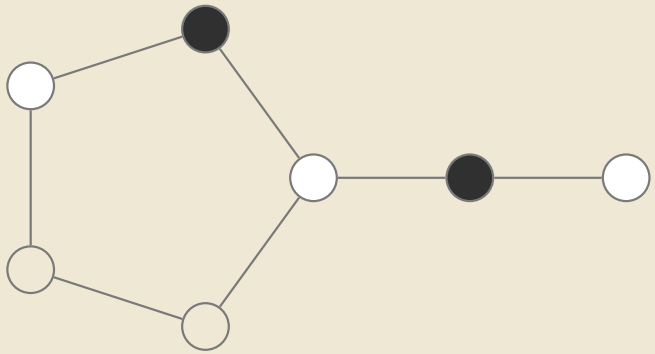

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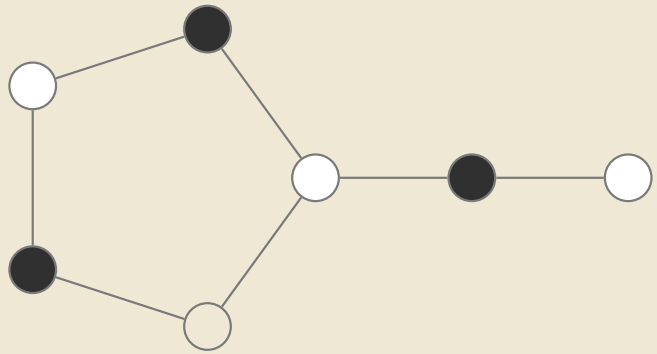

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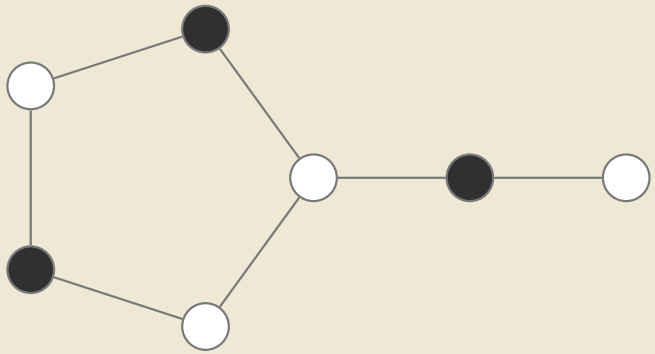

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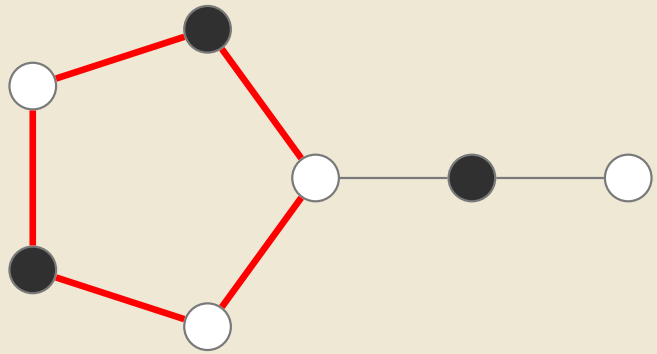

We found a cycle! This cycle must have odd length (otherwise there wouldn't be a conflict).

Fun Facts about Insects

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- ⋆ Some insects, especially butterflies, can exhibit *Gynandromorphism*: having both male and female characteristics.