CSC363 Tutorial #1

Turing machines and stuff

January 18, 2023

I can't believe it's 2023 already

Things you should learn in this tutorial

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- What a Turing machine is, at a high level.
- How to trace the execution of a Turing machine.
- How to program a Turing machine at a low level: defining its states, defining the transition table.

Some administrative stuff

- Quiz 1 will be released on Jan 21, and due on Jan 23. Please remember to complete the quiz! Note that Quer*us sometimes doesn't send you notifications for quizzes.

A bit about myself?

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- Hobbies: Gaming, taking naps at inappropriate times
  - Favourite food: sushi juice
  - D&D alignment: neutral good???? idk
  - horoscope: ummm... you can have my birthday i guess... december 11
  - mbti: hmmmm good question... you can try to guess...
  - facebook: nope :(
  - twi**er: 
  - discord: you can probably find my discord tag by word of mouth
  - favourite word: poggers
  - favourite fried rice topping: rice
  - where i would like to travel to: my fridge
  - most embarassing moment: when i was 11, i stole a connect-four set from a kid at burger king and made the kid cry :(why did i do that
  - favourite emoji: 💀
  - courses I'm TAing this semester: MAT157, CSC263, CSC363
  - favourite music genre: ummmm... can't really say anything specific... mostly edm related subgenres like breakcore, speedcore, happy hardcore, or dnb, something like that?
  - favourite historical person: nice try fbi
  - what i usually eat: pomeloes in the winter, watermelon in the summer
  - number of posters in my room: 1
  - furthest i've driven a car: 0 km
  - best gaming achievement: completing all super meat boy achievements, including the no death runs
  - how old i feel: simultaneously 15 years old and 63 years old
  - most recent book: tuesdays by morrie! it's a nice book...

What's a Turing Machine?

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1.4 ** Turing Machines

Definition 1.4.1. (Turing) A *Turing machine (automatic machine, a-machine)* M includes a two-way infinite *tape* divided into *cells*, a *reading head* which scans one cell of the tape at a time, and a finite set of internal *states* $Q = \{q_0, q_1, \dots, q_n\}$, $n \geq 1$. Each cell is either blank (B) or has written on it the symbol 1. In a single step the machine may simultaneously: (1) change from one state to another; (2) change the scanned symbol s to another symbol $s' \in S = \{1, B\}$; and (3) move the reading head one cell to the right (R) or left (L). The operation of M is controlled by a partial map $\delta : Q \times S \to Q \times S \times \{R, L\}$ (which may be undefined for some arguments).

The interpretation is that if $(q, s, q', s', X) \in \delta$ then the machine M in state q, scanning symbol s, changes to state q', replaces s by s', and moves to scan one square to the right if $X = R$ (or left if $X = L$). The map δ viewed as a finite set of quintuples is called a *Turing program*. The *input* integer x is represented by a string of $x + 1$ consecutive 1's (with all other cells blank). The Turing machine is pictured in Figure 1.1.

We begin with M in the *starting state* q_1 scanning the leftmost cell containing a 1, called the *starting cell*. If the machine ever reaches the *halting state* q_0, after say s steps, then we say M *halts* and the *output* y is the total number of 1's on the tape. (Note that $f(x) = \max\{x + 1, s\}$ bounds the maximum distance from the starting cell to any cell which is either scanned or contains an input symbol. Hence the determination of y is effective.)

We may assume that M never makes any further moves after reaching state q_0, i.e., that the domain of δ contains no element of the form (q_0, s). We say that M *computes* the partial function ψ provided that $\psi(x) = y$

Figure: Soare's definition of a Turing Machine

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The definition is hard to penetrate! We'll give some analogies and

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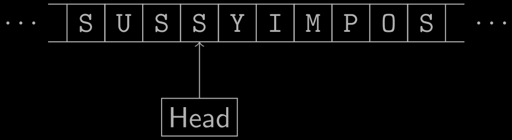
The TM has infinite memory (yay!). However, the memory is difficult to access, because it is in the form of cells on a linear "tape", with only one read/write head. The read/write head can only move left/right one cell at a time...



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The "processor" of a Turing machine is just a DFA. The TM reads one character from its read/write head, and based on the current state, determines the following behaviour:

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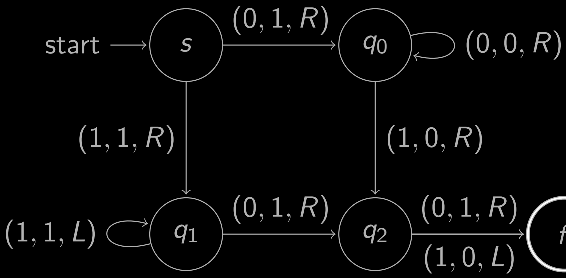
- What to write back to the cell (if anything).
- Which direction (left or right) to move the read/write head.
- The next state of the DFA.



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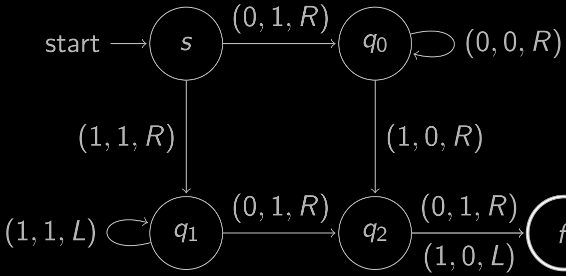

Consider the above DFA, for example. Suppose the TM's processor was in state s.

- If a '0' is read through the read/write head, the following happens:
  - The '0' is overwritten with a '1'.
  - The read/write head moves one cell to the right.
  - The processor transitions to state q_0.
- If instead a '1' is read through the read/write head:
  - The '1' is overwritten with a '1' (i.e. the tape doesn't change).
  - The read/write head moves one cell to the right.
  - The processor transitions to state q_1.

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Some more notes (also on worksheet):

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- Turing Machines can receive string inputs. On input *w* (*w* being some string), place *w* on the tape (at the read/write head) before executing. For example, the TM should look as follows after accepting the input 'CHUNGUS':

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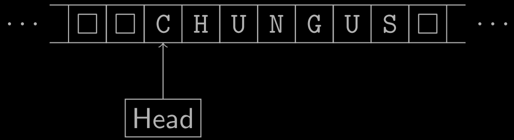

- Each Turing Machine has a designated finish state. For us, the finishing state is called 'f'. Upon reaching the finishing state, execution stops, and whatever string remains on the tape (excluding □ characters) is called the output.

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Now you can cosplay as a Turing machine! **Task:** Complete Exercise 1 on the worksheet.

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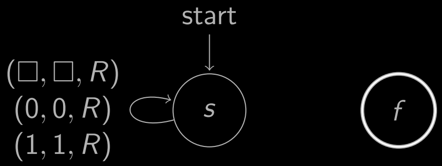

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- $M(w)$ reaches a rejecting state q_{rej}. In this case, we say M rejects w.
- $M(w)$ never reaches an accepting or rejecting state. In this case, we say M loops on w.

Exercise time!

Task: Complete Exercise 2 on the worksheet.

Our favourite website

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Formal definition of a Turing machine

Before giving the formal definition of a TM, please take some time to prepare yourself psychologically. You can have a brief moment to reflect on this photo that I took somewhere in the world.

Figure: Guess where this photo was taken!

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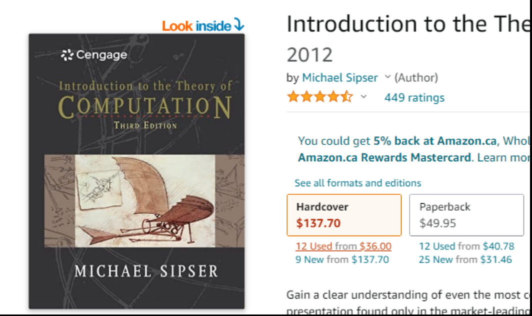
- q_0 is the start state.
- q_{acc} is the accept state.
- q_{rej} is the reject state.

Our favourite book (well, one of them)

The above definition of a Turing machine can be found in Michael Sipser's Introduction to the Theory of Computation. This is one of the reference books that we will use.



wtf

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The TM in Exercise 2 can be formally defined as follows:

- $Q = \{q_0, q_1, q_2, c_1, c_2, r, q_{\mathrm{acc}}, q_{\mathrm{rej}}\}$;
- $\Sigma = \{0, 1\}$;
- $\Gamma = \{\square, 0, 1\}$;
- δ is defined with the following table:

(q, l)	(State, Letter, Direction)
$(q_0, 0)$	$(q_1, \square, R)$
$(q_0, 1)$	$(q_2, \square, R)$
$(q_0, \square)$	$(q_{\mathrm{acc}}, \square, L)$
$(q_0, \#)$	doesn't matter!
$(q_1, 0)$	$(q_1, 0, R)$
$(q_1, 1)$	$(q_1, 1, R)$
$\dots$	$\dots$

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Turing machines are a nice way to formalize the idea of an "algorithm"!