pw1

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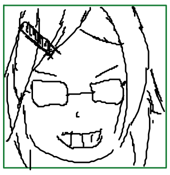
hewwo! owo

Paul Zhang
pol.zhang@utoronto.ca (note the spelling, someone took paul.zhang@mail.utoronto.ca D:)
office hours: Tues 14:30-15:30, same zoom link
a "paragraph" about myself:
uh, hi! ^^ i'm paul, i'm doing a math specialist and a cs minor. i like sushi juice. i can't make this sentence rhyme. :(
p.s. kinda not used to onenote, bear with me here! i'll try my best to make it work tho.



me if i had a proper profile pic
(i cant draw sowwy ;-;)

1) Use the method of completing square to solve the following equation. Consider all the three possibilities which might happen.

$a^2x^2 + 2ax + c = 0$ ($a \neq 0$, a and c are real numbers.)

$a^2x^2+2ax+c=0$

$x^2+\frac{2}{a}x+\frac{c}{a^2}=0 \quad (a^2\neq 0)$

$x^2+\frac{2}{a}x+\frac{1}{a^2}-\frac{1}{a^2}+\frac{c}{a^2}=0$

$(x+\frac{1}{a})^2-\frac{1}{a^2}+\frac{c}{a^2}=0$

$(x+\frac{1}{a})^2=\frac{1-c}{a^2}$

if $\cancel{\frac{1-c}{a^2}<0}$ $c>1$, no solution.

if $\cancel{\frac{1-c}{a^2}=0}$ $c=1$, one solution.

if $\cancel{\frac{1-c}{a^2}>0}$ $c<1$, two solutions.

$x+\frac{1}{a}=\frac{\pm\sqrt{1-c}}{a}$

$x=\frac{-a\pm\sqrt{1-c}}{a}$

Recall Q. Formula proof...

$ax^2+bx+c=0$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

$(x+y)^2=x^2+2xy+y^2$

$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}=0$

$2y=\frac{b}{a}$ $\quad y=\frac{b}{2a}$

$(x+\frac{b}{2a})^2-\frac{b^2}{4a^2}+\frac{c}{a}=0$

$y^2=\frac{b^2}{4a^2}$

$\frac{1-c}{a^2}<0$

$1-c<0$

$c>1$

2) If $a, b > 0$, prove that $\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{ab}$.

Method 1.

Rough Work: $\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{ab}$ ← want to prove

$(\frac{1}{a}+\frac{1}{b}>0)$

$\rightarrow 2 \leq \sqrt{ab}\left(\frac{1}{a}+\frac{1}{b}\right)$

$1 \leq \sqrt{ab}\,\frac{\frac{1}{a}+\frac{1}{b}}{2}$

$\frac{1}{\sqrt{ab}} \leq \frac{\frac{1}{a}+\frac{1}{b}}{2}$

$\sqrt{\frac{1}{a}\cdot\frac{1}{b}} \leq \frac{\frac{1}{a}+\frac{1}{b}}{2}$

↑ true by AGM

Proof: $\sqrt{\frac{1}{a}\cdot\frac{1}{b}} \leq \frac{\frac{1}{a}+\frac{1}{b}}{2}$ (by AGM)

$\frac{1}{\sqrt{ab}} \leq \frac{\frac{1}{a}+\frac{1}{b}}{2}$

$1 \leq \sqrt{ab}\left(\frac{\frac{1}{a}+\frac{1}{b}}{2}\right)$ $(\sqrt{ab}>0)$

$2 \leq \sqrt{ab}\left(\frac{1}{a}+\frac{1}{b}\right)$ $(2>0)$

$\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{ab}$ $(\frac{1}{a}+\frac{1}{b}>0)$

Method 2.

Rough work: $\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{ab}$

$2 \leq \sqrt{ab}\left(\frac{1}{a}+\frac{1}{b}\right)$

$4 \leq ab\left(\frac{1}{a}+\frac{1}{b}\right)^2$

$4 \leq ab\left(\frac{1}{a^2}+\frac{2}{ab}+\frac{1}{b^2}\right)$

$4 \leq \frac{b}{a}+2+\frac{a}{b}$

$2 \leq \frac{b}{a}+\frac{a}{b}$

$2 \leq \frac{b^2+a^2}{ab}$

$2ab \leq b^2+a^2$

$0 \leq b^2-2ab+a^2 = (b-a)^2$

3) If a, b and c are positive real numbers and $2a+3b=c$, find the maximum of ab in terms of c.

Hint: $\frac{2a+3b}{2} \geq \ldots$

$\sqrt{xy} = \frac{x+y}{2}$ iff $x=y$

$\frac{2a+3b}{2} \geq \sqrt{2a\cdot 3b} = \sqrt{6ab}$

$\frac{2a+3b}{2} = \sqrt{6ab}$ iff $2a=3b$

$\frac{c}{2} \geq \sqrt{6ab}$

$2a+3b=c$
$2a-3b=0$

$\frac{c^2}{4} \geq 6ab$

$\frac{c^2}{24} \geq ab$