some topics for today!
- triangle inequality
- sets, set operations, cartesian product of sets
- proving set (in)equalities

1) If a, b and c are three real numbers, prove that $|a - c| \leq |a - b| + |b - c|$.

$$|a - c| = |a - b + b - c| = |(a-b) + (b-c)| \leq |a-b| + |b-c|$$

triangle inequality applied to $(a-b)$ and $(b-c)$

2) Is it true that $(A \times A) \setminus (B \times B) = \boxed{(A \setminus B) \times (A \setminus B)?}$ Explain.

protip: when dealing with "is it true or false" questions, it usually helps to try out a few examples first.

False, $A \times A = \{(a,b) : a \in A, b \in A\}$

$A = \{1, 2\}$ $A \times A = \{(1,1), (1,2), (2,1), \cancel{(2,2)}\}$ $A \setminus B = \{1\}$

$B = \{2, 3\}$ $B \times B = \{(2,2), (2,3), (3,2), (3,3)\}$

$(A \times A) \setminus (B \times B) = \{(1,1), (1,2), (2,1)\}$ $(A\setminus B) \times (A\setminus B) = \{(1,1)\}$

not equal.   * "$A = B$" means "$A \subseteq B$ and $B \subseteq A$"

3) True or false? Justify your answer.
$A \times A \subset B \times B$ if and only if $A \subset B$.

`⊂` is the same as `⊆`

$$A \times A \subseteq B \times B \iff A \subseteq B$$

Pf ($\Rightarrow$) Suppose $A \times A \subseteq B \times B$.
WTS $A \subseteq B$
"want to show"

Showing $A \subseteq B$ {
let $a \in A$. $(a, a) \in A \times A$
Since $A \times A \subseteq B \times B$, $(a, a) \in B \times B$
$(a,a) \in B \times B = \{(x,y) : x \in B, y \in B\}$
this means $a \in B$
So $A \subseteq B$.
}

($\Leftarrow$) Suppose $A \subseteq B$.
WTS $A \times A \subseteq B \times B$.
Let $z \in A \times A$.
$z \in A \times A = \{(x,y) : x \in A, y \in A\}$.
$z = (x, y)$ for some $x \in A, y \in A$
$x \in A \Rightarrow x \in B$ } Since $A \subseteq B$.
$y \in A \Rightarrow y \in B$

therefore $z \in B \times B = \{(a,b) : a \in B, b \in B\}$
So $A \times A \subseteq B \times B$.