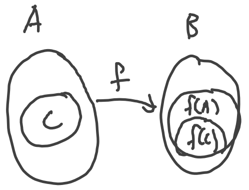
w3

Monday, May 24, 2021 6:37 PM

my computer died! D: i'll try to get it repaired within the next week.
in the mean time, you'll have to deal with this browser-based version of onenote. :(
also, sorry for the bad microphone quality today!
(i'll also be late uploading this week's notes)

things covered in this tutorial:
- more sets, of course!
- images of sets under functions
- there's also one question on fields
- hmm... if we're done early, i could go over some quiz questions on request?



1) (a) Let $f : A \longrightarrow B$ be a function, and $C, D \subseteq A$. Prove that $f(C) \setminus f(D) \subseteq f(C \setminus D)$.

Recall: $f(C) = \{f(x) : x \in C\}$

Pf: Let $y \in f(C) \setminus f(D)$. $y \in f(C)$, so $y = f(x)$ for some $x \in C$. $\longrightarrow x \notin D$ (or else $y = f(x)$ but $y \neq f(x)$ since $x \in D$) $\longrightarrow x \in C \setminus D$

$y \notin f(D)$, so $y \neq f(z)$ for any $z \in D$.

since $y = f(x)$, $y \in f(C \setminus D)$

(b) Prove or disprove: if $f : A \longrightarrow B$ be a function, and $C, D \subseteq A$ then $f(C) \setminus f(D) = f(C \setminus D)$. (false)

try some examples!

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

$C = (-\infty, 0)$ $D = (0, \infty)$

$f(C) = (0, \infty)$ $f(D) = (0, \infty)$

$f(C) \setminus f(D) = \emptyset \neq f(C \setminus D) = f(C) = (0, \infty)$

$f : \mathbb{R} \to \mathbb{R}$ $f(x) = 1$

$C = \{-1\}$ $D = \{1\}$

$f(C) = \{1\}$ $f(D) = \{1\}$ $f(C) \setminus f(D) = \emptyset$

$f(C \setminus D) = f(C) = \{1\}$

2) Is the set $\mathbb{R}^2$, with addition and multiplication defined below a field? Explain.

$\{(x,y) : x, y \in \mathbb{R}\}$

$(a, b) + (c, d) = (a + c, b + d)$ $(a, b).(c, d) = (ac, bd)$

Hint: what is the additive identity and multiplicative identity in this "field"?

bonus: is there a way to define '+' and '.' over R^2 so that it is a field? (not that this is relevant for MAT102, but it's a nice thing to know)

if this were a field, it must have an additive identity (x_0, y_0) s.t.

$(x_0, y_0) + (a, b) = (a, b)$ (for any $(a,b) \in \mathbb{R}^2$).

$(x_0 + a, y_0 + b) = (a, b)$

$x_0 + a = a \Rightarrow x_0 = 0$

$y_0 + b = b \Rightarrow y_0 = 0$

So the additive identity must be $(0, 0)$.

it must also have a multiplicative identity (x_1, y_1)

$(x_1, y_1) \cdot (a, b) = (a, b)$ (for any $(a,b) \in \mathbb{R}^2$)

$x_1 \cdot a = a$ if $a = 1$, $x_1 \cdot 1 = 1 \Rightarrow x_1 = 1$

$y_1 \cdot b = b$ if $b = 1$, $y_1 \cdot 1 = 1 \Rightarrow y_1 = 1$.

So the multiplicative identity is $(1, 1)$.

Consider $(0, 1)$. This is not equal to the additive identity $(0, 0)$,

hence it must have a multiplicative inverse (x, y) s.t. $(0, 1) \cdot (x, y) = (1, 1)$.

$0 \cdot x = 1$

this is impossible!

Bonus: yes!

$(a, b) + (c, d) = (a + b, c + d)$

$(a, b) \cdot (c, d) = (ac - bd, ad + bc)$

$(a + bi) + (c + di) = (a + b) + (c + d)i$

complex numbers!