w4

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mmm... hello! my computer is no longer sick! yayy
but i (paul) am sick :(
p.s. i'm sorry about the quiz marks ;-;

topics:
- statements
- quantifiers
- negating statements
- truth tables

1) Consider the following two statements:

$R =$ "For any real number x, there is a real number y, such that $x + y < 1$."
$S =$ "There is a real number y, such that for any real number x, we have $x + y < 1$."

(a) Write both statements using logic symbols.

$R =$ "$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x+y<1)$"

$S =$ "$(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x+y<1)$"

(b) Write the negation of R and S. Use the the logic symbols, but do not use the symbols $\neg$ and $\not<$.

$\neg R =$ "$(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x+y \geq 1)$"

$\neg S =$ "$(\forall y \in \mathbb{R})(\exists x \in \mathbb{R})(x+y \geq 1)$"

* note: $(\forall x \in \mathbb{R})$ does not change into $(\exists x \notin \mathbb{R})$
$(\exists y \in \mathbb{R})$ does not change into $(\forall y \notin \mathbb{R})$

(c) Is R a true or a false statement? Is S a true or a false statement? Explain.

R is true.

Proof of $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x+y<1)$

R

Let $x \in \mathbb{R}$. Choose $y = -x$.

Then $x+y = x+(-x) = 0 < 1$.

S is false.

Proof of $(\forall y \in \mathbb{R})(\exists x \in \mathbb{R})(x+y \geq 1)$

$\neg S$

Let $y \in \mathbb{R}$. Choose $x = -y+1$.

Then $x+y = -y+1+y = 1 \geq 1$.

* Conclusion: changing the order of quantifiers may change the truth value!

2) Let P, Q , R and S be four statements. If $[(P \wedge Q) \vee R] \Rightarrow (R \vee S)$ is a false statement, what must be the truth values of P, Q , R and S? Why?

$$\underbrace{[(P \wedge Q) \vee R]}_{T} \Rightarrow \underbrace{(R \vee S)}_{F}$$

$(R \vee S)$ is false

$\boxed{R \text{ is false}}$, S is false.

$$\underbrace{(P \wedge Q)}_{\text{true}} \vee \boxed{R} \text{ is true}$$

$P \wedge Q$ is true

P is true, Q is true.

3) Construct the truth table of the following statements.

$$(P \Rightarrow Q) \Rightarrow (P \wedge Q)$$

P	Q	$P \Rightarrow Q$	$P \wedge Q$	$(P \Rightarrow Q) \Rightarrow (P \wedge Q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F