Problem 1
Prove that $\log_{30}(45)$ is irrational.

Problem 2

1. Compute $\gcd(932, 656)$ using the Euclidean algorithm.
2. Compute $\gcd(144, 89)$ using the Euclidean algorithm. What do you notice?

Problem 3
Recall *Bézout's Identity*: Let $a, b \in \mathbb{Z}$, not both zero. Then there are $m, n \in \mathbb{Z}$ such that

$$am + bn = \gcd(a, b).$$

1. Compute $\gcd(217, 93)$ using the Euclidean algorithm.
2. Find integers $m, n \in \mathbb{Z}$ such that $217m + 93n = \gcd(217, 93)$, using *back substitution* from the previous subquestion.

Problem 4
Let $a, b \in \mathbb{Z}$, not both zero, and $c \in \mathbb{Z}$. Show that $\gcd(a, b) \mid c$ if and only if there are $m, n \in \mathbb{Z}$ such that

$$am + bn = c.$$