

Problem 1

Is it possible to find $x, y \in \mathbb{Z}$ such that $5x + 11y = 4$?

Problem 2

Let p be prime, and $a \in \mathbb{N}$. Show that $p \mid a^2$ if and only if $p \mid a$.

Problem 3

Let $a, n \in \mathbb{N}$. Show that there is $k \in \mathbb{N}$ such that $ak \equiv 1 \pmod{n}$ if and only if $\gcd(a, n) = 1$.

Problem 4

1. Find an equivalence relation over \mathbb{R} that satisfies the following:
 - The equivalence relation has uncountably infinitely many equivalence classes.
 - Each equivalence class has countably many members.
2. Find an equivalence relation over \mathbb{R} that satisfies the following:
 - The equivalence relation has countably infinitely many equivalence classes.
 - Each equivalence class has uncountably infinitely many members.

Problem 5

Show that 7270324727853158 is not a square number without using a calculator. *Hint: There are no square numbers in the sequence 2, 6, 10, 14, ...*