

**Problem 1**

Let  $A, B$  be subsets of some universal set  $U$ . Prove that  $A \setminus B^C = A \cap B$ .

**Problem 2**

1. Let  $S = \{1, 2, 3\}$ . List all subsets of  $S$ .
2. Let  $S$  be a finite set with  $n$  elements. How many distinct subsets of  $S$  are there?

**Problem 3**

Let  $f : (2, \infty) \rightarrow \mathbb{R}, f(x) = \frac{x+1}{x^2-4}$ . Prove that the range of  $f, f((2, \infty))$ , is  $(0, \infty)$ .

**Problem 4**

Find an example of a function  $f : A \rightarrow B$  and sets  $C, D \subseteq A$  such that  $f(C \cap D) \neq f(C) \cap f(D)$ .

**Problem 5**

Let  $A$  be the “set of all sets”. Define  $S = \{X \in A : X \notin X\}$ . Identify a logical contradiction, assuming the existence of  $S$ .<sup>a</sup>

<sup>a</sup>This contradiction is called **Russell’s Paradox**, which arises due to our informal and loose definition of a “set” as a collection of distinct objects. In this course, you won’t have to worry about such malicious examples of “sets”, but it may be helpful to remember that MAT102 presents a simplified but workable version of set theory.

One way to avoid Russell’s Paradox is to use a more formal axiomatic definition of “set”, such as the **Zermelo-Frankel Axioms (ZF)**. Under the ZF axioms, sets can’t be a members of themselves ( $X \notin X$  for all sets  $X$ ).