Problem 1

Find the range of each of the following functions.

1. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \dfrac{1}{1 + 2^{-x}}$.

2. $f : \mathbb{N} \to \mathbb{Z}$, $f(n) = (-1)^n$.

3. $f : \mathbb{N} \times \mathbb{Z} \to \mathbb{R}$, $f(n, m) = n + m - 2$.

Problem 2

Let A, B, C be sets. Suppose $A \setminus C = B \setminus C$.

1. Give an example of sets A, B, C satisfying the above such that $A \neq B$.

2. Suppose furthermore that $A \cap C = B \cap C$. Show that $A = B$.

Problem 3

Let $f : A \to B$ be a function, and $C \subseteq B$. We define the **preimage** of the set C as

$$f^{-1}(C) = \{x \in A : f(x) \in C\}.$$

(Note that "f^{-1}" should not be confused with the *inverse* of f; the inverse of f might not exist.)

1. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$. Find $f^{-1}((1, 4])$.

2. Ignoring the context of the previous subquestion, suppose $C, D \subseteq B$. Show that
   $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$.

3. Is it true that $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$? Give a proof or counterexample.