Problem 1
For each of the following, find a bijection from A to B.

1. $A = \mathbb{N}$, $B = \mathbb{N} \setminus \{1, 3\}$.
2. $A = \mathbb{N}$, $B = \mathbb{Z}$.
3. $A = (0, \infty)$, $B = \mathbb{R}$.
4. $A = \mathbb{R}$, $B = (-1, 1)$.

Problem 2
Which of the following sets are not countable?

1. $\mathbb{R} \setminus \mathbb{Q}$.
2. $\mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}$.
3. $\mathcal{P}(\mathbb{N})$.
4. $\{f : f \text{ is a function with domain } \mathbb{N} \text{ and codomain } \mathbb{Q}\}$.

Problem 3

- Let $a < b$. Find a bijection from the interval $[a, b]$ to $[0, 1]$.
- Let $c < d$. Find a bijection from the interval $[0, 1]$ to $[c, d]$.
- Conclude that any two closed intervals have the same cardinality.

Problem 4
Compute the power set of $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$. *Hint: You may find it easier to substitute* $A = \emptyset$, $B = \{\emptyset\}$, $C = \{\emptyset, \{\emptyset\}\}$ *before you start.*

Problem 5
Let S be a set. Show that there is an injection from S to $\mathcal{P}(S)$.

Problem 6
Let U be the "set of all sets". Show that U cannot exist, using Cantor's theorem ($|S| < |P(S)|$ for any set S). *Hint: If U were a set, then what is $P(U)$?*